Specification and Verification of Distributed Systems

January 25, 2007

Exercise 5

Which of the following statements hold for arbitrary monoids $\mathbb{M}$ and arbitrary $\alpha, \alpha_1, \alpha_2 \in \mathrm{REX}_{\mathbb{M}}$? In case that a statement is not true for arbitrary monoids, is there a monoid that makes it true?

(a) $L(\alpha \cdot \boldsymbol{\emptyset}) = \emptyset$

(b) $L(\alpha) = \emptyset$ implies $\alpha = \boldsymbol{\emptyset}$

(c) $L(\alpha_1 \cdot \alpha_2) = L(\alpha_2 \cdot \alpha_1)$

(d) $L((\alpha_1 + \alpha_2)^*) = L(\alpha_1^* + \alpha_2)^*$

Exercise 6

Let $\widetilde{\Sigma} = (\{a,b\},\{b,c\},\{d\})$. For any of the following rational expressions α_i of $\mathbb{TR}(\widetilde{\Sigma})$, examine if α_i is star-connected. If not, can you specify a star-connected rational expression $\beta \in \mathrm{REX}_{\mathbb{TR}(\widetilde{\Sigma})}$ such that $\mathcal{L}(\beta) = \mathcal{L}(\alpha_i)$?

$$\alpha_1 = (a \cdot \boldsymbol{\emptyset} \cdot c + d)^*$$

$$\alpha_2 = (a \cdot \boldsymbol{\emptyset}^* \cdot c)^*$$

$$\alpha_3 = (a(c(abc)^*)^*)^* + (a + bc + d)^*$$

$$\alpha_4 = ((a + c)^* + acd)^*$$

Exercise 7

Let $\widetilde{\Sigma} = (\{a,b\},\{b,c\},\{d\})$ and consider the following subsets of $\mathbb{TR}(\widetilde{\Sigma})$:

$$\mathcal{L}_1 = \{\mathcal{T} \in \mathbb{TR}(\widetilde{\Sigma}) \mid \mathcal{T} \text{ is connected}\}$$

$$\mathcal{L}_2 = \{\mathcal{T} \in \mathbb{TR}(\widetilde{\Sigma}) \mid \mathcal{T} \text{ is not connected}\}$$

$$\mathcal{L}_3 = \{(E, \leq, \lambda) \in \mathbb{TR}(\widetilde{\Sigma}) \mid |\{e \in E \mid \lambda(e) = d\}| \text{ is even}\}$$

$$\mathcal{L}_4 = \{(E, \leq, \lambda) \in \mathbb{TR}(\widetilde{\Sigma}) \mid |\{e \in E \mid \lambda(e) = b\}| \text{ is even}\}$$

$$\mathcal{L}_5 = \{(E, \leq, \lambda) \in \mathbb{TR}(\widetilde{\Sigma}) \mid \leq \text{ is a total order}\}$$

For $i = 1, \ldots, 5$, determine

(a) a rational expression $\alpha \in \mathrm{REX}_{\mathbb{TR}(\widetilde{\Sigma})}$ such that $\mathcal{L}(\alpha) = \mathcal{L}_i$.

(b) a sentence $\varphi \in \mathrm{MSO}(\Sigma, \{\leq, \lessdot\})$ such that $\mathcal{L}(\varphi) = \mathcal{L}_i$.