Examen:

Modélisation et vérification des systèmes temporisés, hybrides ou concurrents

— Systèmes concurrents —

15 février 2007

Problem 1

Let $\widetilde{\Sigma} = (\{a, b\}, \{b, c\}, \{d\})$ be a distributed alphabet. Consider the following subsets of Σ^*:

$$L_1 = \{w \in \Sigma^* \mid |w|_a = |w|_c\}$$

$$L_2 = \{w \in \Sigma^* \mid |w|_a + |w|_d \text{ is even}\}$$

$$L_3 = \{uadcv \mid u, v \in \Sigma^*\}$$

For $i = 1, 2, 3$, determine if

(a) $L_i = [L_i]_{\sim_{\widetilde{\Sigma}}}$ (recall that $[L_i]_{\sim_{\widetilde{\Sigma}}} := \{u \in \Sigma^* \mid \text{there is } v \in L_i \text{ such that } u \sim_{\widetilde{\Sigma}} v\}$).

(b) $[L_i]_{\sim_{\widetilde{\Sigma}}}$ is regular.

(c) there is an asynchronous automaton $\mathcal{A}$ over $\widetilde{\Sigma}$ such that $L(\mathcal{A}) = [L_i]_{\sim_{\widetilde{\Sigma}}}$.

(d) there is a product automaton $\mathcal{A}$ over $\widetilde{\Sigma}$ such that $L(\mathcal{A}) = [L_i]_{\sim_{\widetilde{\Sigma}}}$.

(e) there is a locally accepting product automaton $\mathcal{A}$ over $\widetilde{\Sigma}$ such that $L(\mathcal{A}) = [L_i]_{\sim_{\widetilde{\Sigma}}}$.

Justify your answers!

(Recall that, for $w \in \Sigma^*$ and $\sigma \in \Sigma$, $|w|_\sigma$ denotes the number of occurrences of σ in w. For example, $|abaac|_a = 3$ and $|abc|_d = 0$.)

Problem 2

Let $\widetilde{\Sigma} = (\{a, b\}, \{b, c\}, \{d\})$ be a distributed alphabet. For any of the following rational expressions $\alpha_i \in \mathrm{REX}_{\mathbb{TR}(\widetilde{\Sigma})}$ $(i = 1, 2, 3)$, examine if α_i is star-connected. If not, is there a star-connected rational expression $\beta \in \mathrm{REX}_{\mathbb{TR}(\widetilde{\Sigma})}$ such that $\mathcal{L}(\beta) = \mathcal{L}(\alpha_i)$?

$$\alpha_1 = \emptyset \cdot (a \cdot c)^*$$

$$\alpha_2 = (a + c^*)^* \cdot d$$

$$\alpha_3 = (a \cdot (c \cdot b^*)^*)^*$$

Problem 3

Let $Proc = \{1, 2, 3, 4\}$ and let Msg be a singleton set.

(a) Determine a formula $\varphi(x, y) \in \mathrm{MSO}(Act, \{\leq, <_{\mathrm{mes}}\})$ with free variables x and y such that, for any $\mathcal{M} = (E, \leq, \lambda) \in \mathbb{MSC}$ and any interpretation function $\mathcal{I}$,

$$\mathcal{M} \models_{\mathcal{I}} \varphi(x, y) \text{ iff } (\mathcal{I}(x), \mathcal{I}(y)) \in (\leq \cup \leq^{-1})^*.$$

(b) Determine a sentence $\varphi \in \mathrm{MSO}(Act, \{\leq, <_{\mathrm{mes}}\})$ such that

$$\mathcal{L}_{\mathbb{MSC}}(\varphi) = \{\mathcal{M} \in \mathbb{MSC} \mid \mathcal{M} \text{ is connected}\}.$$

Is there a rational expression $\alpha \in \mathrm{REX}_{\mathbb{MSC}}$ such that $\mathcal{L}(\alpha) = \mathcal{L}_{\mathbb{MSC}}(\varphi)$?

(c) Determine a star-connected rational expression $\alpha \in \mathrm{REX}_{\mathbb{MSC}}$ such that

$$\mathcal{L}(\alpha) = \{(E, \leq, \lambda) \in \mathbb{MSC} \mid \text{for any } e \in E,\ \lambda(e) \in \{1!2, 2?1, 2!3, 3?2, 3!4, 4?3\}\}.$$

Is there an equivalent rational expression that is not star-connected?

(d) Determine a star-connected rational expression $\alpha \in \mathrm{REX}_{\mathbb{MSC}}$ such that

$$\mathcal{L}(\alpha) = \{(E, \leq, \lambda) \in \mathbb{MSC} \mid \text{for any } e \in E,\ \lambda(e) \in \{1!2, 2?1, 2!3, 3?2\}\}.$$

Is there an equivalent rational expression that is not star-connected?

Problem 4

Argue that the following problem is undecidable for $Proc = \{1, 2, 3\}$ and a singleton set Msg:

INPUT: MPA $\mathcal{A}$ pver $Proc$ and Msg.

QUESTION: Is $\mathcal{A}$ $\exists B$-bounded for some $B \geq 1$?