Exercice 1

(1)

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\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma \vdash \forall x\ ((P\ x) \Rightarrow (P\ (S\ x)))}\ \text{axiome}}{\Gamma \vdash (P\ (S\ 0)) \Rightarrow (P\ (S\ (S\ 0)))}\ \forall\text{-elim} \qquad \dfrac{\dfrac{\dfrac{}{\Gamma \vdash \forall x\ ((P\ x) \Rightarrow (P\ (S\ x)))}\ \text{axiome}}{\Gamma \vdash (P\ 0) \Rightarrow (P\ (S\ 0))}\ \forall\text{-elim} \qquad \dfrac{}{\Gamma \vdash (P\ 0)}\ \text{axiome}}{\Gamma \vdash (P\ (S\ 0))}\ \Rightarrow\text{-elim}}{\Gamma \vdash (P\ (S\ (S\ 0)))}\ \Rightarrow\text{-elim}}{(P\ 0) \vdash (\forall x\ ((P\ x) \Rightarrow (P\ (S\ x)))) \Rightarrow (P\ (S\ (S\ 0)))}\ \Rightarrow\text{-intro}}{\vdash (P\ 0) \Rightarrow (\forall x\ ((P\ x) \Rightarrow (P\ (S\ x)))) \Rightarrow (P\ (S\ (S\ 0)))}\ \Rightarrow\text{-intro}
$$

où $\Gamma = (P\ 0), \forall x\ ((P\ x) \Rightarrow (P\ (S\ x)))$.
(2) $\lambda x\ \lambda f\ (f\ (S\ 0)\ (f\ 0\ x))$

Exercice 2

(1) non, (2) oui, (3) non, (4) oui, (5) oui, (6) oui.

Exercice 3

(1) $d = (Rec\ 0\ \lambda x\ \lambda y\ (S\ (S\ y)))$
(2) $a = \lambda z\ \lambda k\ < z, j(s\ x\ (d\ z)\ k) >$, $b = \lambda z\ \lambda l\ < (S\ z), i(s\ x\ (S\ (d\ z))\ l) >$
(3) $t = (Rec\ (0, i(refl\ 0))\ \lambda x\ \lambda h\ (\delta\ (\pi_2\ h)\ (a\ (\pi_1\ h))\ (b\ (\pi_1\ h))))$
(4) $\exists y\ (((S\ (S\ (S\ (S\ 0)))) = (d\ y)) \vee ((S\ (S\ (S\ (S\ 0)))) = (S\ (d\ y))))$
(5) 0
(6) $nat \to nat$
(7) $(S\ (S\ 0))$