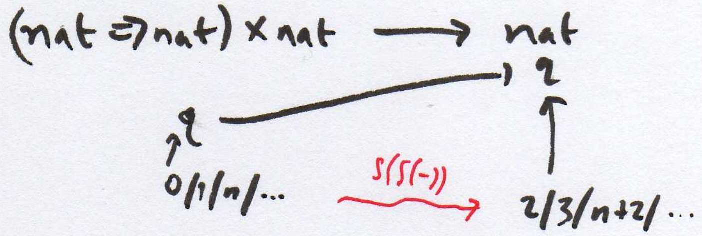
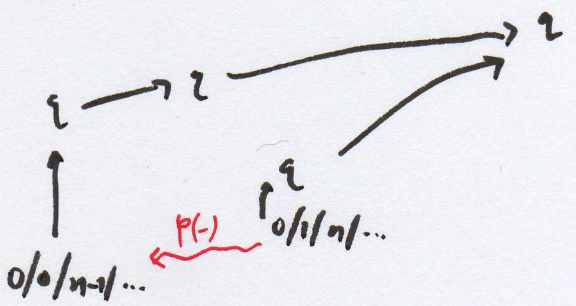
• D_1 : start with $f : nat \to nat,\ x : nat \vdash S(S(f(Px))) : nat$

(1)

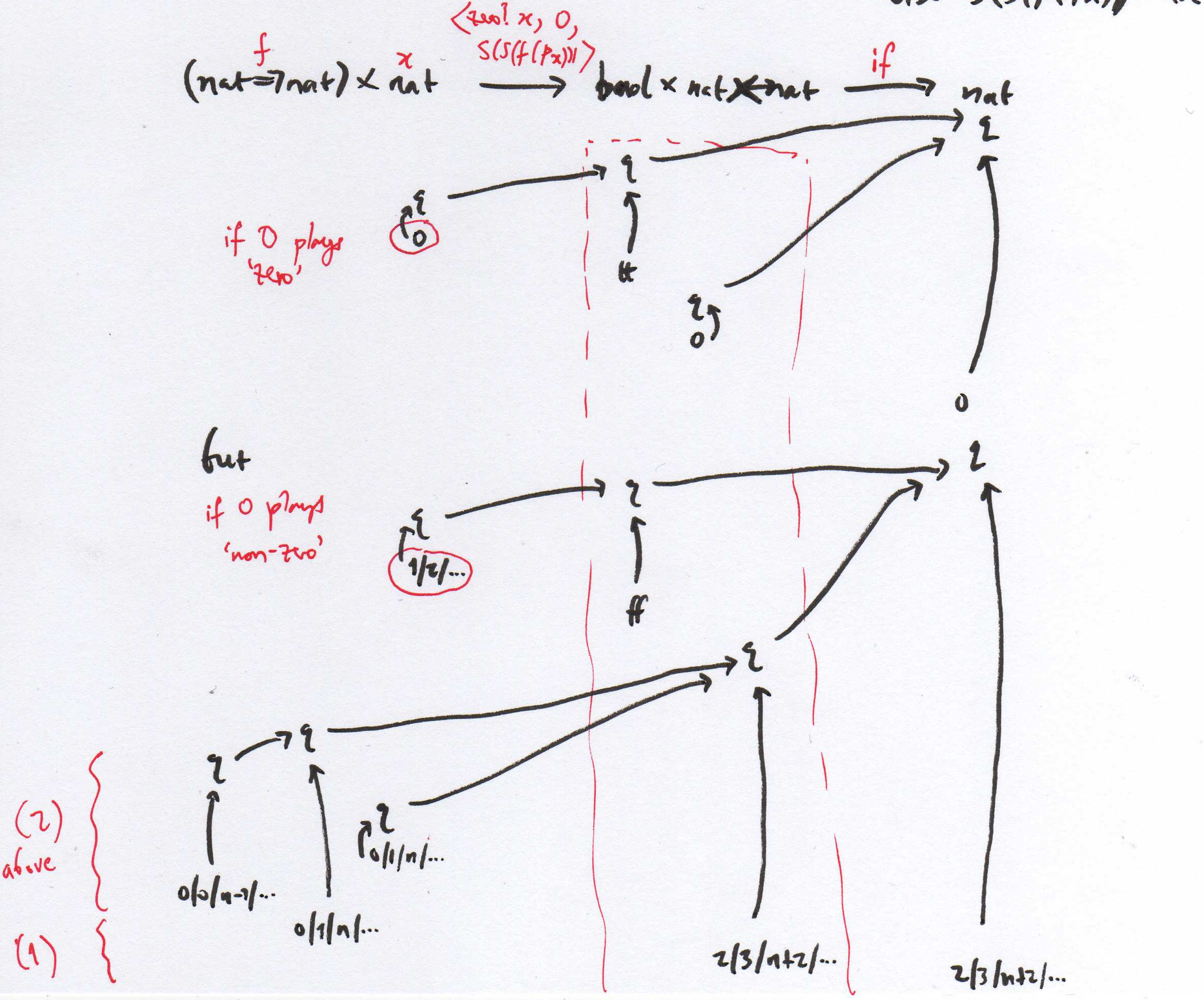


and

(2)

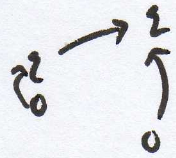


now let's do $f : nat \to nat,\ x : nat \vdash \text{if (zero? } x) \text{ then } 0$
$\text{else } S(S(f(Px))) : nat$

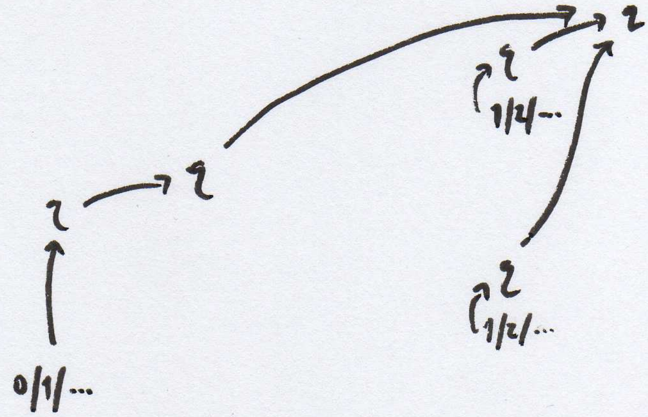


So $f : nat \to nat \vdash \lambda x(-) : nat \to nat \;\triangleq\; F$

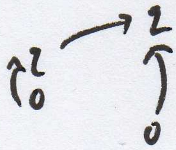
$(nat \Rightarrow nat) \xrightarrow{F} (nat \Rightarrow nat)$



and

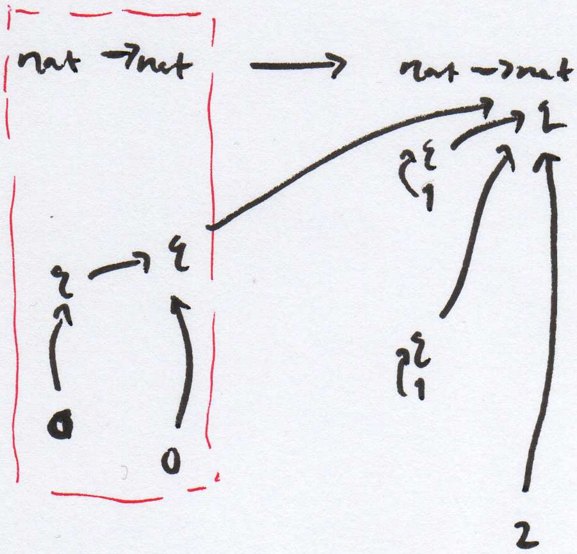


$\leadsto \; 1 \xrightarrow{\bot} nat \Rightarrow nat \xrightarrow{F} nat \Rightarrow nat$



and that's all

$1 \xrightarrow{F(\bot)} nat \to nat \longrightarrow nat \to nat$



etc. to calculate $(\mathrm{fixpt}\ \lambda f(D_1)) : nat \to nat$

Interactions

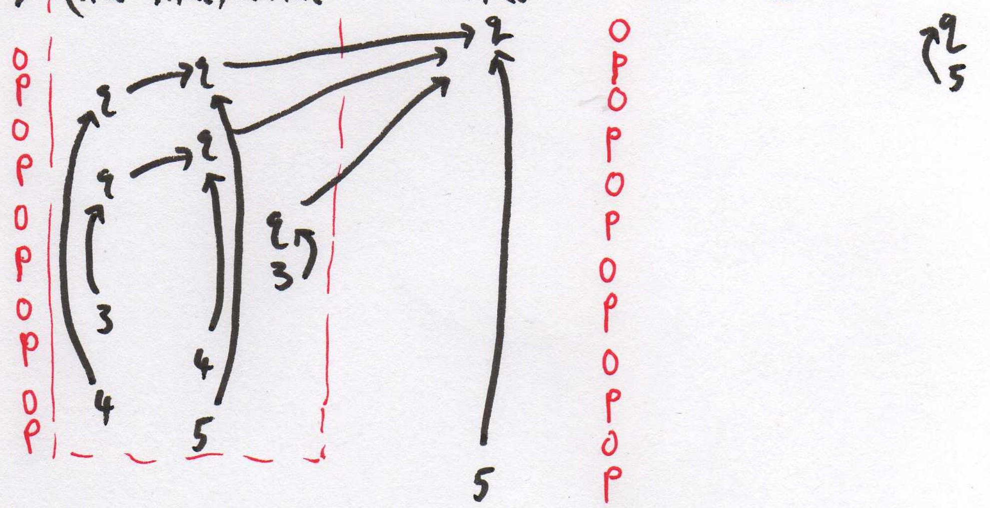
- $\{\lambda f x\,(f\,(f\,x))\}\,(\lambda z(\mathrm{succ}\,z))\;3 \;:\; \mathrm{nat}$

suffices to calculate

$f : \mathrm{nat}\to\mathrm{nat},\ x : \mathrm{nat} \vdash f(f\,x) : \mathrm{nat}$

$\langle \lambda z(\mathrm{succ}\,z), 3\rangle \;;\; (f(f\,x)) \;:\; \mathrm{nat}$

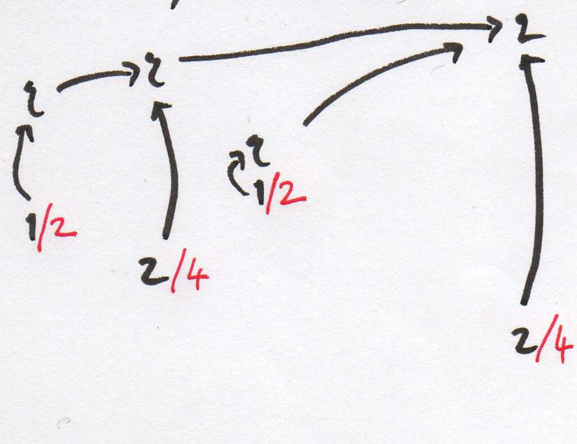
$\mathbb{1} \longrightarrow (\mathrm{nat}\Rightarrow\mathrm{nat})\times\mathrm{nat} \longrightarrow \mathrm{nat} \;\rightsquigarrow\; \mathbb{1} \longrightarrow \mathrm{nat}$



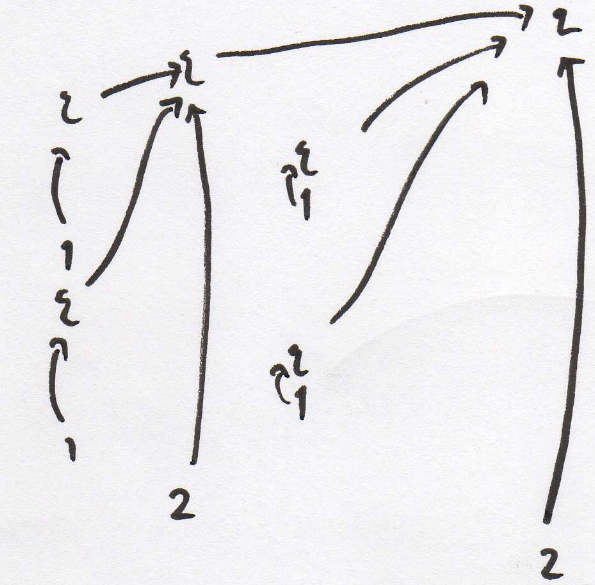
- $\cancel{\Lambda(\mathrm{nat}\Rightarrow\mathrm{nat})}\quad(\mathrm{fixpt}\;\;\lambda f(D_2))\,\tfrac{1}{2} \;:\; \mathrm{nat}$

$\langle \mathrm{fixpt}\,\lambda f(D_2), \tfrac{1}{2}\rangle$

$\mathbb{1} \longrightarrow (\mathrm{nat}\Rightarrow\mathrm{nat})\times\mathrm{nat} \xrightarrow{\mathrm{eval}} \mathrm{nat}$



- $1 \xrightarrow{\langle \mathrm{fix}_{nat}\, \mathcal{F}(D_1),\, 1\rangle} (nat \Rightarrow nat) \times nat \xrightarrow{eval} nat$



$\bullet\ f : (\mathbb{B} \Rightarrow \mathbb{B}) \Rightarrow \mathbb{B} \vdash f(\lambda x(f(\lambda y(x))))$

- Suffices to calculate $N ; \Lambda^{-1}(K_{x\mathbb{B}})$: $1 \xrightarrow{N} ((\mathbb{B} \Rightarrow \mathbb{B}) \Rightarrow \mathbb{B}) \longrightarrow \mathbb{B}$

and $1 \xrightarrow{N} ((\mathbb{B} \Rightarrow \mathbb{B}) \Rightarrow \mathbb{B}) \xrightarrow{K_y} \mathbb{B}$