2D Triangulations

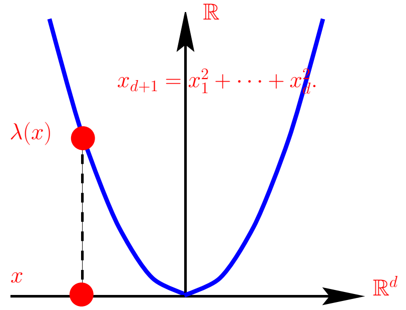
- Localisation
- Delaunay triangulation
- Enumeration algorithm
- Tutte Barycentric Theorem

Notes

Delaunay Triangulation

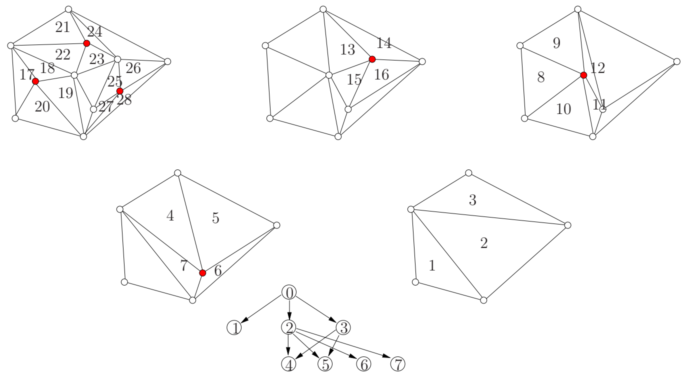
- $S \subset \mathbb{R}^d$ un ensemble de n points.
- $\mathcal{R}$ l'ensemble des parties $R \neq \emptyset$ de S telles qu'il existe une d-sphère *vide* circonscrite à R.
- Pour $R \in \mathcal{R}$ on appelle *cellule de Delaunay* de R et on note $D(R)$ l'enveloppe convexe de R.
- L'ensemble D des cellules de Delaunay est appelé la triangulation de Delaunay de S.

Triang. Delaunay & Enveloppe Convexe



- La triang. Delaunay de S est la proj. env. conv. inf. $\lambda(S)$
- Statut actuel du problème de l'enveloppe convexe.

Localisation



$\#$ faces $= 2n - 2 - h$, $\quad \#$ arêtes $= 3n - 3 - h$, $\quad \#$ sommets $= n$.

Couverture et Structure de Localisation

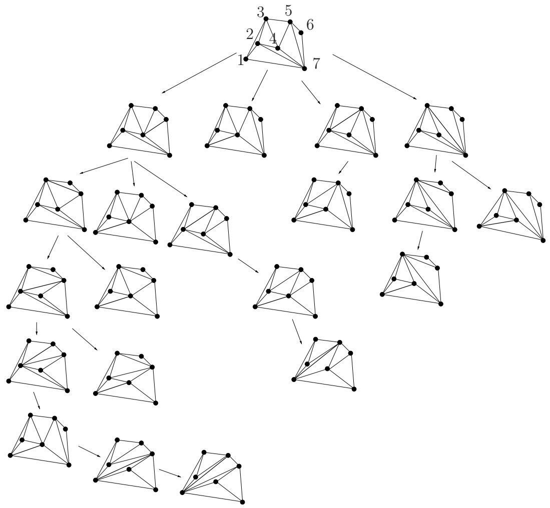
Soit T une triangulation d'un ensemble P de n points du plan. Une triangulation T' d'un sous-ensemble P' de P est appelée une couverture de T de paramètres $c \in \mathbb{N}$ et $\alpha \in \mathbb{R}$ si

1. P et P' ont la même enveloppe convexe
2. $|P'| \leq \alpha |P|$
3. tout triangle de T' est sécant à au plus c triangles de T.

Theorem 1. *Soit c un entier ≥ 6 et soit $\alpha = 1 - \frac{1}{c+1}(1 - \frac{6}{c+1})$. Il existe une couverture T' de T de paramètres c et α calculable en temps linéaire.* □

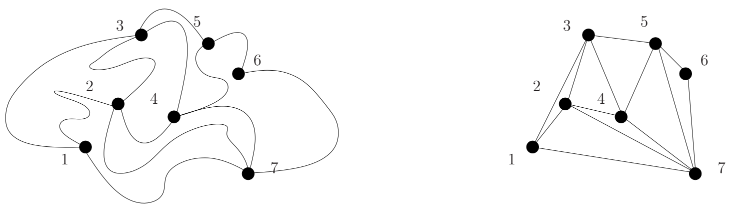
- Suite de triangulations $T = T_1, T_2, \ldots, T_{O(\log n)}$
  - T_{i+1} est une couverture de T_i de paramètres 11 et $\frac{23}{24}$.
  - Chaque triangle de T_{i+1} pointe sur les au plus 11 triangles de T_i qui lui sont sécants.
- Taille de la structure $O(n)$.
- Temps de requête $O(\log n)$.

Enumeration



Notes

Tutte Barycentric Theorem



Theorem 2 (Tutte, 1962). *Let $G = (V, E)$ be a 3-connected, planar graph on n vertices and let $C_e = (V_e, E_e)$ be a distinguished cell of G. Let $\omega : E \setminus E_e \to \mathbb{R}^+$ be an assignement of positive weights to the* <u>*internal*</u> *edges and let $p : V_e \to \mathbb{R}$ be an assignement of positions to the* <u>*external*</u> *vertices such that $p(C_e)$ is a strictly convex polygon. Then*

1. *There are unique positions $p(v)$ for the internal vertices (i.e., vertices of $V \setminus V_e$) such that all internal vertices are in equilibrium, i.e.,*

$$\sum_{\{v,w\} \in E} \omega_{v,w}(p(v) - p(w)) = 0$$

2. *All cells of G are then realized as non-overlapping convex polygons.* □

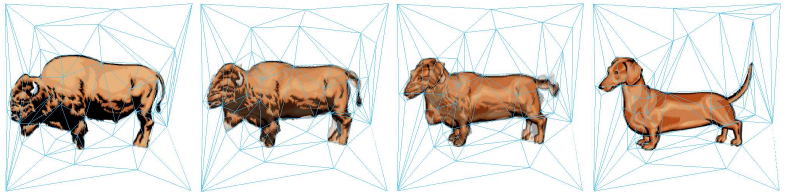
Tutte Barycentric Theorem

Let $G = (V, E)$ be a 3-connected, planar graph on n vertices and let $C_e = (V_e, E_e)$ be a distinguished cell of G. An assignment $p : V \to \mathbb{R}^2$ of positions in $\mathbb{R}^2$ for the vertices of G is called a *good representation* for G, if the two following properties are satisfied

1. p is a convex planar embedding of C_e.
2. for all $v \in V \setminus V_e$ the image of v under p belongs to the relative interior of the convex hull of the images under p of the neighbors of v.

Theorem 3. *Let p be a good representation of a 3-connected, planar graph G. Then p is a planar embedding of G in which all interior cells are realized as non-overlapping convex polygons.* □

Isotopy



Bibliographie

References

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