Academic year 2007/08 - Course on Concurrency: 2^{nd} partial examination

You may consult the slides of the lectures. No other document or electronic device is allowed. Answers should be formulated in French or English, and preferably in a rigorous and sharp style.

Please write the solutions to the two parts in separate sheets.

First part

Exercise 1 (Expressivity, 6.5 points) *Recall that the language generated by P, L(P), is the set of all sequences generated from the finite-maximal labelled transitions of P. More precisely,*

$$L(P) = \{s \in \mathcal{L}^* \mid \exists Q : P \stackrel{s}{\Longrightarrow} Q \wedge \forall \alpha \in \mathcal{L} \cup \{\tau\} : Q \stackrel{\alpha}{\not\longrightarrow}\}$$

where $\mathcal{L}$ denote the set of visible actions in CCS

- **Exercise 1.1:** Give a CCS! (CCS with replication) process P that generates the non-regular language $\{a^n b^n c \mid n \geq 0\}$

- **A Solution:** Consider the process P below:

$$\begin{array}{rcl} P & = & (\nu k_1, k_2, k_3, u_b)(\ \overline{k_1} \mid \overline{k_2} \mid Q_a \mid Q_b \mid Q_c) \\ Q_a & = & !k_1.a.(\overline{k_1} \mid \overline{k_3} \mid \overline{u_b}) \\ Q_b & = & k_1.!k_3.k_2.u_b.b.\overline{k_2} \\ Q_c & = & k_2.(c \mid u_b.DIV) \end{array}$$

  where $DIV = !\tau$. One can verify that $L(P) = \{a^n b^n c\}$.

Now recall that P is *weakly terminating* iff P generates at least one sequence, i.e., $L(P) \neq \emptyset$. Also recall that P is *termination-preserving* iff whenever $P \stackrel{s}{\Longrightarrow} Q \stackrel{\tau}{\longrightarrow} R$: If Q is weakly terminating then R is weakly terminating.

- **Exercise 1.2:** Prove that termination-preserving CCS! processes can generate non context-free languages. Hint: Since context-free languages are closed under intersection with regular languages, it suffices to give a P such that $L(P) \cap a^*b^*c^* = \{a^n b^n c^n \mid n \geq 0\}$.

- **A Solution:** Take

$$P = (\nu k, u)(\overline{k} \mid !k\, a.(\overline{k} \mid \overline{u})) \mid k.!u\,(b \mid c))$$

One can verify that P is termination-preserving. Furthermore, $L(P) \cap a^*b^*c^* = a^nb^nc^n$, hence $L(P)$ is not a CFL since CFL's are closed under intersection with regular languages.

Exercise 2 (Probability, 4.5 points) *Consider the following process P:*

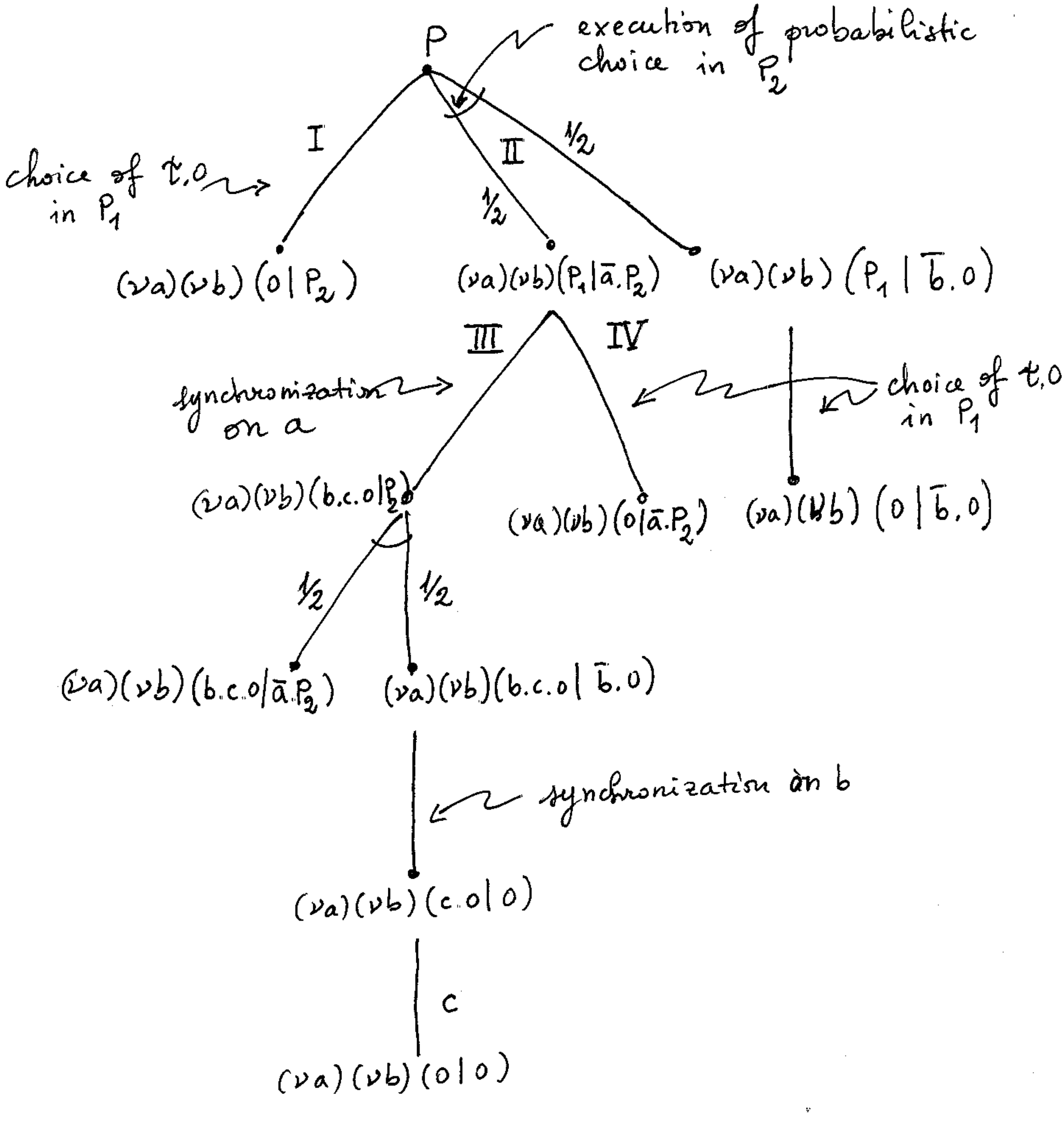
$$(\nu a)(\nu b)((a.b.c.0 + \tau.0) \mid (\mathbf{let}\ X = (\bar{a}.X \oplus_{1/2} \bar{b}.0)\ \mathbf{in}\ X))$$

Assume that a, b and c are pairwise different.

Exercise 2.1 Draw the graph of P

Solution

Let P_1 be the process $a.b.c.0 + \tau.0$ and P_2 be the process $\mathbf{let}\ X = (\bar{a}.X \oplus_{1/2} \bar{b}.0)\ \mathbf{in}\ X$. The graph generated by P is the following:



Exercise 2.2 How may different schedulers we have for P? Motivate your answer.

Solution

There are 3 different schedulers:

- The scheduler σ_1, which selects the transition **I**,
- the scheduler σ_2, which selects the transition **II** and then **III**,
- the scheduler σ_3, which selects the transition **II** and then **IV**.

Exercise 2.3 What is the probability that c will be executed, under the different schedulers?

Solution

The probability of performing c is

- 0 under σ_1,
- 1/4 under σ_2,
- 0 under σ_3