Exercises, 17 november 2006

CCS: bisimulation

1. Here are the specifications of unary and binary semaphores:

$$K_1 = p.v.K_1 \qquad\qquad K_2 = p.K_2' \qquad K_2' = p.v.K_2' + v.K_2$$

   Prove that $K_1 \parallel K_1 \sim K_2$.

2. Let $K = a.K$. Prove that $K \parallel K \sim K$.

3. Prove that $(\boldsymbol{\nu} c)(K_1 \parallel K_2) \sim H$, where $K_1 = a.\bar{c}.K_1$, $K_2 = b.c.K_2$, and $H = a.b.\tau.H + b.a.\tau.H$.

4. Prove these equivalences:

$$\begin{array}{rcl}(\boldsymbol{\nu} a)(\tau.\bar{b} \parallel c) & \approx & \bar{b} \parallel c \\ (\boldsymbol{\nu} a)(b.\bar{a} \parallel a.c) & \approx & b.c \\ \tau.P \parallel Q & \approx & \tau.(P \parallel Q)\end{array}$$

5. Expain why $\tau.(\tau.a + b) + \tau.b \not\approx \tau.a + \tau.b$. Is it true that $K \approx a$, where $K = \tau.K + a$?

CCS: equational reasoning

1. Consider the following processes, where the channel names a, b, c, d, e, f are all distinct:

$$\begin{array}{rcl}P & = & (\boldsymbol{\nu} b)(a.b.c.\mathbf{0} \parallel d.\bar{b}.e.\mathbf{0}) \\ Q & = & (\boldsymbol{\nu} f)(a.f.c.\mathbf{0} \parallel d.\bar{f}.e.\mathbf{0}) \\ R & = & a.c.\mathbf{0} \parallel d.e.\mathbf{0}\end{array}$$

   Show that $P = Q$ while $P \neq R$, where $=$ denotes the axiomatization for weak bisimilarity.

pi-calculus: reductions

1. Show that the term

$$S = (\boldsymbol{\nu} x)((x(u).u(y).u(z).\bar{y}\langle z\rangle.\mathbf{0} \;\Big|\Big|\; x(t).t(w).t(v).\bar{v}\langle w\rangle.\mathbf{0}) \;\Big|\Big|\; !(\boldsymbol{\nu} s)\bar{x}\langle s\rangle.\bar{s}\langle a\rangle.\bar{s}\langle b\rangle.\mathbf{0})$$

   reduces to $\bar{a}\langle b\rangle.\mathbf{0} \parallel \bar{b}\langle a\rangle.\mathbf{0}) \parallel (\boldsymbol{\nu} x)(!(\boldsymbol{\nu} s)\bar{x}\langle s\rangle.\bar{s}\langle a\rangle.\bar{s}\langle b\rangle.\mathbf{0})$. Let S_1 be the first process S reduces to in the sequence above: show that $S \xrightarrow{\tau} S_1$.

pi-calculus: bisimulation

1. Show that $(\boldsymbol{\nu} z)(\bar{z}\langle a\rangle \parallel z(w).\bar{x}\langle w\rangle) \approx \bar{x}\langle a\rangle$.

2. Show that $(\boldsymbol{\nu} z)\bar{x}\langle y\rangle.P \approx \bar{x}\langle y\rangle(\boldsymbol{\nu} z)P$.

3. Using the up-to structural congruence and up-to context proof technique, show that

$$!!\bar{x}\langle y\rangle.y(z) \approx !\bar{x}\langle y\rangle.y(z)\ .$$

pi-calculus: data structures

1. Recall the implementation of booleans as pi-calculus processes seen in the lectures. Define a process $EQUIV$ that represents the operation of equivalence on truth-values. Show that

$$(\boldsymbol{\nu} b, c)(EQUIV\lfloor a, b, c\rfloor \;\Big|\Big|\; TRUE\lfloor b\rfloor \;\Big|\Big|\; FALSE\lfloor c\rfloor) \approx FALSE\lfloor a\rfloor\ .$$

pi-calculus: asynchronous communication

1. Show how to encode diadic asynchronous communication using monadic asynchronous communication, and check that $[[\bar{x}\langle y_1, y_2\rangle \parallel x(z_1, z_2).R]] \rightarrow^* [[R\{^{y_1,y_2}/_{z_1,z_2}\}]]$.

pi-calculus: types

1. Propose a type assignment for channels of the processes used to represent truth-values (first using simple types, then using i/o types).